BRIEF REPORT

Producing oscillatory decisions

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Mathematical models in neuroscience have led to the formulation of new theories of cognition. Several models have focused on equilibrium-converging cognitive activities but human and artificial cognition are frequently dynamic. Here we present some of the models that produce dynamic behavior of cognition, specifically those that produce temporal oscillations in decision-making.

Keywords: cognition; decision-making; choice selection; dynamical system; oscillations

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Introduction

Cognitive activities are multidimensional and multifaceted. However, we can still predict, up to some level, the outcomes of the complex cognitive activities using mathematical models. Mathematical modeling is one of the techniques in formulating abstract representation of the interaction among the factors that shape cognition.

There are many cognitive activities in human or artificial brain. For simplicity, we here focus on decision-making activities that involve conflicting choices. Decision-making spans multiple scales, such as from genetic and organismal level to community-level [1]. Various mathematical models of decision-making employed naïve and static representations. Although sensitivity analysis can be done, some of these models consider equilibrium or fixed outcome. Classical examples of such models are [2, 3, 4, 5, 6, 7] (i) Weighted sum model; (ii) Weighted product model; (iii) Analytic hierarchy process; (iv) ELECTRE; (v) TOPSIS; (vi) Analytic Network Process; (vii) PROMETHEE and GAIA; and (viii) Dominance-based rough set approach.

Several mathematical models applied dynamical systems, such as involving difference equations and ordinary differential equations [1,8,9,10]. If for all cases the preference of a choice effectively counter-balances the preference of the other choices (e.g., in symmetric decision-making system), then the outcome may converge to a stable equilibrium state. However, oscillations may also arise as outcome of the dynamic models. The oscillations can be simple or complex, which are occasionally chaotic [11]. The cycles in the oscillations could sometimes be phase-locked (synchronous to each other) or sometimes heteroclinic [9, 12]. Oscillations can be a result of circular tug-of-war (e.g., repressilator-type/intransitive competitive antagonism [1, 13, 14]) among the conflicting choices during decision-making. Oscillatory behavior may characterize various cognitive behaviors, such as cyclic dominance, indecisiveness, inconsistency of preference, randomization (e.g., rock-paper-scissors), inherent complexity of decision process, risk perception, or psychiatric disorder [8, 9, 15].

In this paper, we consider decision-making that involves conflicting choices or alternatives. These choices are competing to each other. However, note that similar to ecological interactions, choices may not always undergo competition. In some cases, choices are neutral (the choices have no negative or positive effect to the other choices) or cooperative (preference of a choice mutually increases the

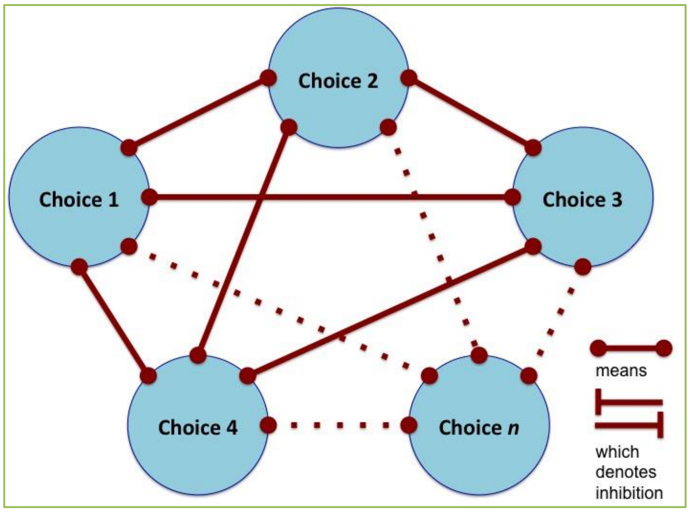

Figure 1. **Network of conflicting choices.**

preference of the other choices). Decision-making can be considered as a game-theoretic process that may involve different kinds of strategies [16, 17].

Mathematical models and Discussion

Let us consider the network of conflicting choices in Figure 1. We assume in our models that choice preference is affected by the worth of the other choices. A higher worth value means that the choice is more preferred.

Several models can be employed to represent the temporal dynamics of decision-making. Here the temporal worth of a choice is affected by the temporal worth of the other choices. These worth values could be fixed (equilibrium-converging) or fluctuating. The following are examples of simple models that show qualitative dynamics of choice selection ($i=1,2,\dots,n$; n is the number of choices).

(i) *Lotka-Volterra competition model* [9]:

$$\frac{\Delta X_i}{\Delta t} = X_i + hX_i\left(r_i - \sum_{j=1}^{n} \alpha_{ij} X_j\right).$$

Alternatively, this non-polynomial model can be used:

$$\frac{\Delta X_i}{\Delta t} = X_i + hX_i\left(r_i - \sum_{j=1}^{n} \frac{\alpha_{ij} {X_j}^c}{K_{ij} + {X_j}^c}\right).$$

(ii) *Multistable decision switch model with self-stimulation* [1]:

$$\frac{\Delta X_i}{\Delta t} = X_i + h\left(\frac{\beta_i {X_i}^c}{K_i + \sum_{j=1}^{n} \alpha_{ij} {X_j}^c} - d_i X_i + g_i\right).$$

(iii) *Decision switch model without self-stimulation* [18]:

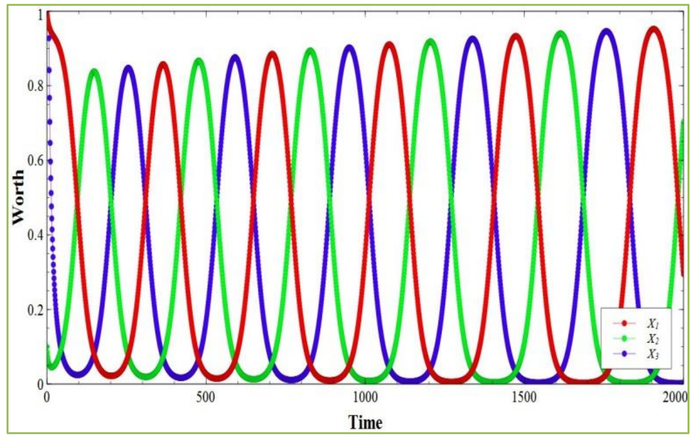

Figure 2. **An example of decision outcome using Lotka-Volterra type competition model with small *h*.** Parameter values are h=0.01 and $r_1=r_2=r_3=5$. Initial condition is (1, 0.1, 2). The matrix [α_{ij}] is $\begin{bmatrix} 5 & 10 & 0.1 \\ 0.1 & 5 & 10 \\ 10 & 0.1 & 5 \end{bmatrix}$.

$$\frac{\Delta X_i}{\Delta t} = X_i + h\left(\frac{\beta_i}{K_i + \sum_{j=1, j\neq i}^{n} \alpha_{ij} {X_j}^c} - d_i X_i + g_i\right).$$

(iv) *Linear model using a transformed comparison matrix* [19]:

$$\frac{\Delta X_i}{\Delta t} = X_i + h\left(\sum_{j=1}^{n} \alpha_{ij} X_j\right) \text{ where } \alpha_{ij} = -\alpha_{ji}.$$

The variable X_i denotes the normalized worth of choice i. The parameter h is the time step during decision process. The matrix [α_{ij}] represents the strength of conflict among the choices. If the worth of a choice increases, then the worth of the other choices could decrease. For the definition of the other parameter values and detailed information about the model, see the associated literature [1, 9, 18, 19].

In the Lotka-Volterra competition model, the conflict in the preference of choices is seen in the terms $-\sum_{j=1, j\neq i}^{n} \alpha_{ij} X_j$. Notice that these terms have negative influence in the growth of X_i's worth. In the decision switch models, the conflict in the preference of choices is represented by $\sum_{j=1, j\neq i}^{n} \alpha_{ij} {X_j}^c$ in the denominator of the growth term. That is, as X_j, $j\neq i$ increases, the growth term of X_i decreases nonlinearly. On the other hand, the linear model using a transformed comparison matrix has a different form of showing conflict of choices. In this model, the conflict is reflected in the matrix of a_{ij}. For further details about this linear model, see model 2 in the associated literature [19].

Figures 2 to 4 show example dynamics arising from Lotka-Volterra competition model (Figs. 2 and 3) and from decision switch model with self-stimulation (Fig. 4). Notice that the oscillations exhibit cyclic dominance, that is, only one choice is dominant for a certain time period and a

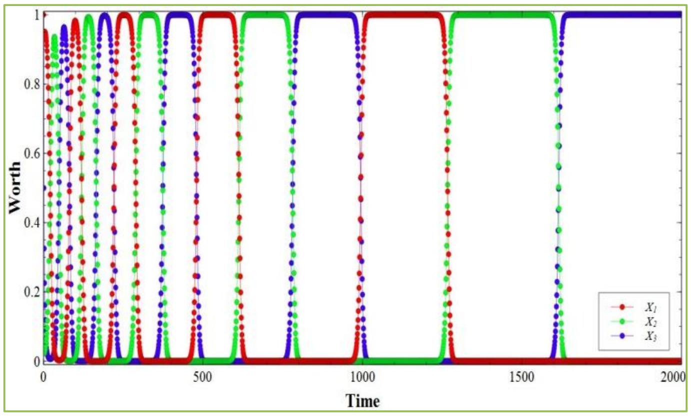

Figure 3. **An example of decision outcome using Lotka-Volterra type competition model with larger *h*.** Parameter values are h=0.05 and $r_1=r_2=r_3$=5. Initial condition is (1, 0.1, 2). The matrix [α_{ij}] is $\begin{bmatrix} 5 & 10 & 0.1 \\ 0.1 & 5 & 10 \\ 10 & 0.1 & 5 \end{bmatrix}$.

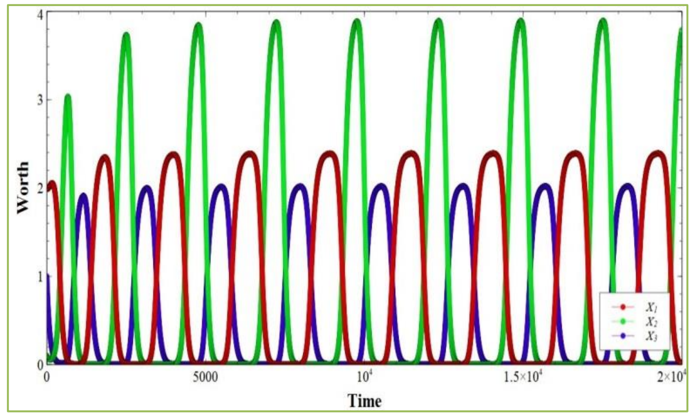

Figure 4. **An example of decision outcome using the decision switch model with self-stimulation.** Parameter values are h=0.2, $\beta_1=\beta_2=\beta_3$=1, K_1=5.2, K_2=5.1, K_3=5, d_1=0.1, d_2=0.11, and d_3=0.09. Initial condition is (2, 0.1, 1). The matrix [α_{ij}] is $\begin{bmatrix} 2 & 10 & 0.01 \\ 0.01 & 1 & 12 \\ 11 & 0.01 & 3 \end{bmatrix}$.

dominant choice is perpetually replaced by a new dominant choice. This cyclic dominance is akin to arms race competition, specifically the Red Queen dynamics [20]. There are other oscillatory dynamics, not only limited to Figures 2-4, which can be simulated using the models. The outcomes of a model depend on the combination of parameter values.

In Lotka-Volterra competition model and decision switch model with self-stimulation, sustained oscillations could arise because of an asymmetric [a_{ij}] such as a matrix [a_{ij}] characterizing a 'repressilator' interaction [13, 14]. In fact, it was reported that having a negative-feedback interaction loop is a good condition for generating oscillations [21, 22]. Consequently, the structure of the decision interaction network (e.g., Fig. 1) can be considered as one of the criteria in analyzing the outcome of choice selection.

The outcome of choice selection can be controlled by regulating the structure of decision interaction network, the formulation of the dynamical system model, and the combination of the parameter values. For future studies, we can investigate the effect of some parameters on decision-making. For example, we can study how the time step *h* affects the behavior of oscillations (e.g., see Figure 2 and 3). Another topic for future research is to investigate the outcome of decision-making when there is hierarchical choice selection similar to the Analytic Hierarchy Process [19]. We hypothesize that, in some cases, hierarchical choice selection may result in chaotic behavior.

In this paper, we have discussed deterministic models but decision-making could be stochastic or fuzzy, and often involves delay. Uncertainties and delay could also result in oscillations [13, 23, 24]. We have focused on simple dynamic models, but in reality, decision-making may not be simple and may be part of a larger decision-making activity. Multi-criteria decision outcomes are often incorporated in an optimization process [25, 26].

Decision-making, specifically choice selection, is one of the activities developed by multifaceted factors including the environment. The study of choice selection is interdisciplinary, involving neurobiology, neuroeconomics, social science, and many other fields. Here we aim to spur more investigations to examine non-equilibrium dynamics of decision-making, which usually happen in our everyday lives [1, 9, 27, 28, 29, 30, 31]. Oscillations may have negative (e.g., psychiatric disorder) or positive implications. A positive implication is oscillations generate temporal diversity of outcomes that could strategically reduce the risks and effects of antagonism [20].

Conflict of interest statement

The authors declare no conflicting interests.

Author contribution

JFR conceived the study. JFR and MKAG draft the manuscript. All authors read and approved the final manuscript.

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