Determining the Optimal Distribution of Bee Colony Locations to Avoid Overpopulation Using Mixed Integer Programming

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ABSTRACT

This paper aims to find the best location of bee colonies in a certain area in such a way that each colony will have enough food resources. Mixed Integer Programming, which is already an established tool in optimization, is used to achieve the said goal.

The output of this paper is a general model that determines the optimal distribution of bee colonies for any size and orientation of a field with diverse kinds of plants. As a prototype example, the best relocation of stingless bee (*Trigona biroi, Friese*) colonies in a small community is determined.

Keywords: hive location, overpopulation, mixed integer programming, optimal distribution

INTRODUCTION

Two of the vital needs of bees are nectar and pollen. Honey is produced by bees from the nectar of blossoms, while pollens are gathered from the anther of the flower. The strength of the bee colonies depends largely on the availability of nectar and pollen.

There is an apparent overlap in the pollen and nectar sources of various bee species in the Philippines, as shown in the studies on *Apis mellifera* (Payawal et al, 1986; 1989; 1991), *Apis cerana* (Tilde et al, 2003), and *Trigona biroi* (Barile and Cervancia, 1995). This condition may lead to competition, especially when the resources are scarce. This can be further aggravated by the prevailing changes in climate patterns that negatively affect phenology, and consequently, bee populations. Otis (1991) and Dino (2004) observed the impact of competition of *Apis mellifera* on colony establishment.

Some beekeepers encounter problems, such as slow down in the rate of bee-product production, due to the competition within the area where they keep bee colonies. Competition may occur among nearby bee colonies. The area of competition is usually in a circular band with radius close to the average flight distance. As the hives become further apart, competition decreases.

To minimize competition, hives may be relocated to various sites, with consideration of the abundance of bee plants. The site must have a good source of nectar, located within the maximum flight distance of the bees. Aside from the nectar, availability of pollen sources must be observed. Beekeepers may use artificial beehives to transport colonies from field to field. Initially, the bees may panic and suffer stress upon relocation, but enough water and food will make the bees survive. Moreover, in relocating the bee colonies, beekeepers must consider the physical characteristics and habits of the species, such as body size (Kuhn-Neto et al, 2009).

This paper will attempt to solve the problem of overpopulation using optimization. In mathematics, mathematical programming refers to choosing the best element from some set of available alternatives. In Industrial Engineering, Integer Programming is used for facility location (Haug, 1985; Schilling et al, 1993). In the case of minimizing the overpopulation of bees, the best location of the bee colony will be chosen from all the possible locations.

MODEL FORMULATION

Construction of a *graph* of the field where the beehives will be distributed is needed prior to the formulation of the mathematical program. Construction of graphs is helpful in visualizing the physical setup of the field.

A graph is composed of nodes and edges. Two kinds of nodes will be considered – the possible relocation sites of the beehives and the clusters of plants. The distance between a relocation site and a plant cluster should be less than or equal to the maximum flight distance of the bee species to be considered connected. This assumption about the connection (edges) is employed to reduce the possibility of flying outside the maximum capability just to search for food; and this will reduce the stress of the bees. Clustering of plants depends on the distribution of the food sources in the area.

Each bee colony may have different strengths. In this paper, strength is defined as the number of foragers of the colony over a certain reference number. The carrying capacity of a plant cluster is the number of foragers that the plant cluster can accommodate over the set reference number. For example, if the reference number is 100 foragers and the plant cluster can accommodate 200 foragers, then the carrying capacity of that plant cluster is 2. The preference of the beekeeper over the location sites can also be considered by giving each site a priority weight.

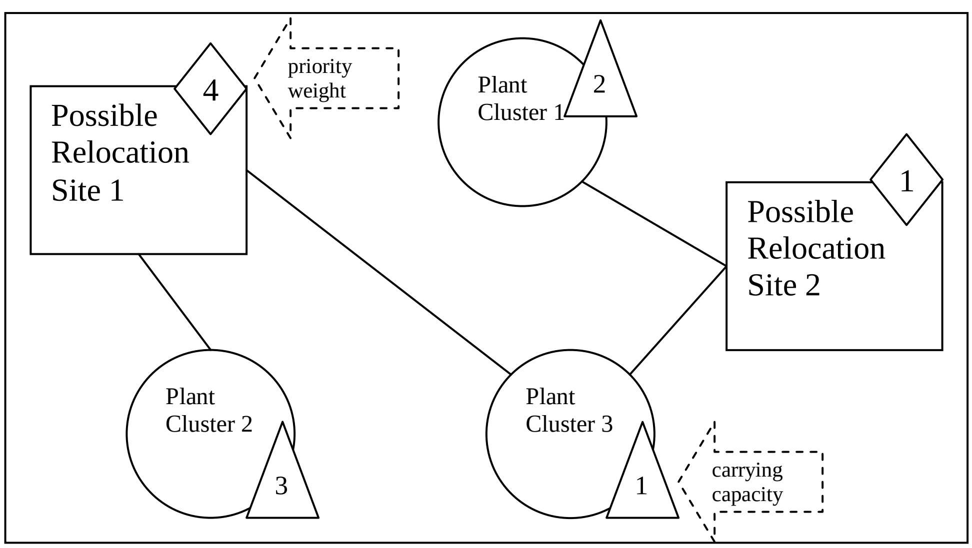

Figure 1. A sample graph of a field

The mathematical program has the following assumptions: bee colonies are non-migratory, a beehive contains one colony only, the input and output values are deterministic, the resources have constant carrying capacity, and the strength of each bee colony does not change for a certain period of time. For future research, the carrying capacity and the colony strength can be made probabilistic. The model also gives an optimal solution even if it is impossible to avoid overpopulation. The mathematical program is given below:

Definition of Variables :

$m \sim$ number of relocation sites

$p \sim$ number of plants or plant clusters

$n \sim$ number of colonies

$X_i \sim$ strength of colony to be relocated at site i; $X_i \in \mathbb{R}^{\oplus}$

$X_{ij} \sim$ fraction of X_i that can be accomodated by plant cluster j

$Y_j \sim$ carrying capacity of plant cluster j; $Y_j \in \mathbb{R}^{\oplus}$

$w_i \sim$ priority weight given to site i; $w_i \in \mathbb{R}^{+}$

$$Z_{ki} = \begin{cases} 1 & \text{ifcolony } k \text{ will be located at site } i \\ 0 & \text{otherwise} \end{cases}$$

$b_k \sim$ strength of colony k; $b_k \in \mathbb{R}^{+}$

$M \sim$ penalty weight given to minimize E_i's; any big positive real number

$E_i \sim$ amount of overpopulation at site i, in terms of colony strength

$j \in C_i \sim$ plant cluster j is connected to site i

$i \in F_j \sim$ site i is connected to plant cluster j

Objective Function: $Maximize \sum_{i=1}^{m} (w_i X_i - ME_i)$

subject to

Constraint 1: $\sum_{\substack{j=1 \\ j \in C_i}}^{p} X_{ij} - X_i = 0 \quad \forall\ i = 1, 2, \ldots, m$

Constraint 2: $\sum_{\substack{i=1 \\ i \in F_j}}^{m} X_{ij} - Y_j \leq 0 \quad \forall\ j = 1, 2, \ldots, p$

Constraint 3: $\sum_{k=1}^{n} b_k Z_{ki} - E_i - X_i \leq 0 \quad \forall\ i = 1, 2, \ldots, m$

Constraint 4: $\sum_{i=1}^{m} Z_{ki} = 1 \quad \forall\ k = 1, 2, \ldots, n$

The objective function represents the goal of maximizing the number of colonies to be relocated at available sites, considering beekeeper's preference, with the minimum degree of overpopulation. Constraint 1 represents the distribution of the strengths of colonies relocated at a site connected to the connected plant clusters. Constraint 2 represents the contribution of a site (in terms of strengths) to the carrying capacity of the connected plant cluster.

Constraint 3 shows how the strengths will be relocated to the sites with the assurance that the colony will not be subdivided into parts. Constraint 4 assures that the colony will have unique relocation site.

Model Testing on a Hypothetical Scenario

Consider a small field community with three available relocation sites, six plant clusters and six colonies of *Trigona biroi* Friese. The maximum flight distance of *Trigona biroi* is set at 500m. Assume that the beekeeper gives "4" as priority weight to relocation site 1, and the reference number is 100 foragers. The following table shows the other input values:

Table 1. Strengths of colonies

	Strength
Colony 1	1
Colony 2	2
Colony 3	½
Colony 4	3
Colony 5	¼
Colony 6	4

Table 2. Carrying capacities of plant clusters

	Capacity
Cluster 1	1
Cluster 2	2
Cluster 3	2
Cluster 4	½
Cluster 5	5
Cluster 6	1

The following graph shows the physical setup of the field environment.

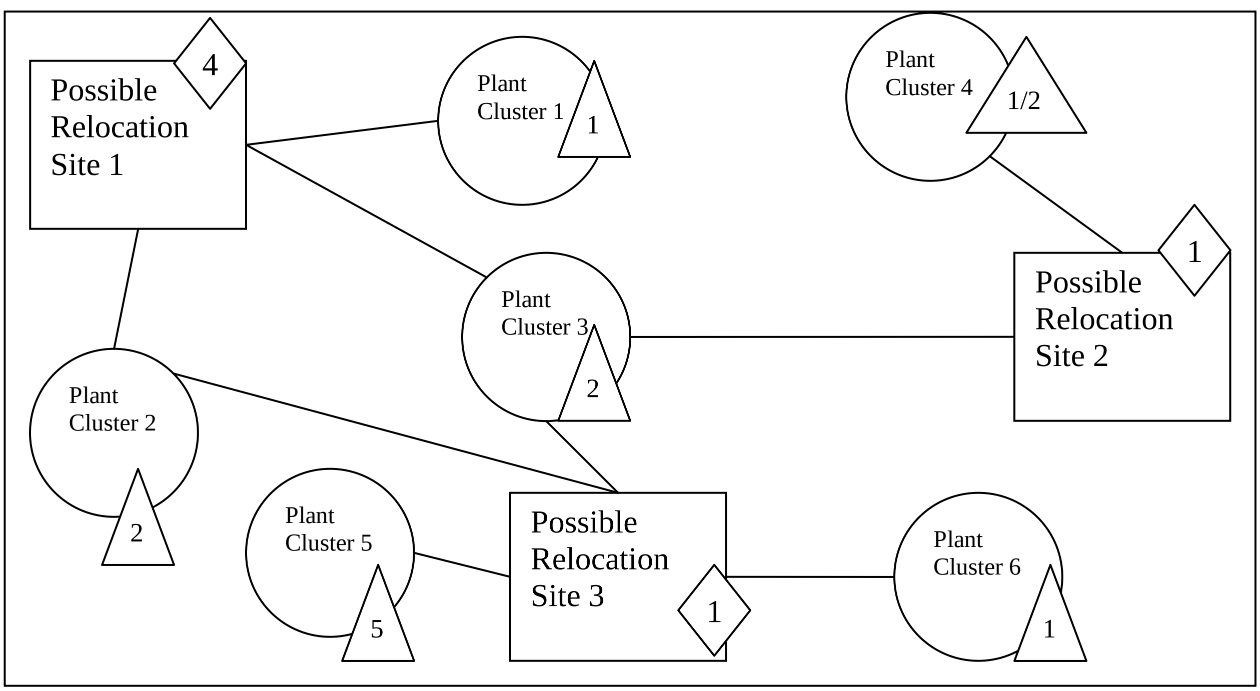

Figure 2. Graph of the field for the hypothetical scenario

The resulting Mixed-Integer program for the hypothetical scenario is as follows:

$$
\begin{aligned}
&Max\ 4X_1 + X_2 + X_3 - 1000(E_1 + E_2 + E_3) \\
&subject\ to \\
&\qquad X_{11} + X_{12} + X_{13} - X_1 = 0 \\
&\qquad X_{23} + X_{24} - X_2 = 0 \\
&\qquad X_{32} + X_{33} + X_{35} + X_{36} - X_3 = 0 \\
&\qquad X_{11} \le 1 \\
&\qquad X_{12} + X_{32} \le 2 \\
&\qquad X_{13} + X_{23} + X_{33} \le 2 \\
&\qquad X_{24} \le 0.5 \\
&\qquad X_{35} \le 5 \\
&\qquad X_{36} \le 1 \\
&\qquad Z_{11} + 2Z_{21} + 0.5Z_{31} + 3Z_{41} + 0.25Z_{51} + 4Z_{61} - E_1 - X_1 \le 0 \\
&\qquad Z_{12} + 2Z_{22} + 0.5Z_{32} + 3Z_{42} + 0.25Z_{52} + 4Z_{62} - E_2 - X_2 \le 0 \\
&\qquad Z_{13} + 2Z_{23} + 0.5Z_{33} + 3Z_{43} + 0.25Z_{53} + 4Z_{63} - E_3 - X_3 \le 0 \\
&\qquad Z_{11} + Z_{12} + Z_{13} = 1 \\
&\qquad Z_{21} + Z_{22} + Z_{23} = 1 \\
&\qquad Z_{31} + Z_{32} + Z_{33} = 1 \\
&\qquad Z_{41} + Z_{42} + Z_{43} = 1 \\
&\qquad Z_{51} + Z_{52} + Z_{53} = 1 \\
&\qquad Z_{61} + Z_{62} + Z_{63} = 1
\end{aligned}
$$

Mixed-Integer Program can be solved using any Operations Research software. As a result of the above mathematical program, colonies 1, 3, 4 and 5 should be relocated at site 1, and colonies 2 and 6 should be relocated at site 3. Site 1, 2 and 3 can accommodate 5, ½ and 6 strengths of colonies, respectively. The solution has no indication of overpopulation.

When the carrying capacities of the plant clusters were changed to ½, 1, 1, ¼, 2½ and ½, respectively, overpopulation cannot be avoided. Site 1, 2 and 3 can contain 2½, ¼ and 3 strengths of colonies, respectively. Total overpopulation is equal to 5 strengths of colonies, i.e. 500 foragers will have difficulty foraging due to overpopulation. Colonies 3, 4 and 6 can be located at site 1, colony 5 can be located at site 2, and colonies 1 and 2 can be located at site 3. This is the optimal solution to the problem given that the carrying capacities were changed.

CONCLUDING REMARKS

Bees play an important role in nature. As pollinators, bees help in the production of flowering crops and in the maintenance of plant biodiversity. Like any animals, bees are threatened by overpopulation. This overpopulation is caused not by continuously growing number of individuals but by the decreasing area of habitat. The resources are limited for a certain number of bee colonies only.

To minimize overpopulation, man should act. One of the techniques that men can do is to optimally distribute bee colonies, especially when the total strength of colonies exceeds the carrying capacity of the environment. The mathematical model presented in this paper is useful in attaining this aim.

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