OPTIMAL DEPLOYMENT OF COMMUNITY POLICE FORCE

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Introduction

Police officers are generally the heroes of peace and order. It is their mission to provide security among the constituents of a community. However, lack of resources hinders the police departments to carry out their services efficiently and effectively.

Operations Research uses techniques that could optimize the level of police services given limited resources, specifically the limited number of manpower. Manpower size is dependent on the budget allocated to the security force.

Optimizing the level of police services entails: (1) *minimizing the emergency response time*; (2) *increasing security coverage and visibility*; (3) *reducing personnel fatigue caused by rationalized shift-schedules*; and importantly, (4) *determining the best possible number of police officers to be allocated given the budget constraint*.

According to Geroliminis, et al. (nd.), multiple methods exist in allocating police personnel. There are deterministic models (used for planning purposes) and stochastic/probabilistic models (used in operational scenarios). Mitchell (1972) uses P-median Model to determine the police patrol areas in Anheim, California. Backup Coverage Model maximizes the number of areas covered more than once (Daskin, 1983). Larson's M/M/n Hypercube Queuing Model examines probabilistic models for emergency systems and finds the optimal deployment pattern for a predetermined set of patrol areas (Larson, 1975 and 1978).

The Model

Goal Programming, which is a Multi-Criteria Decision-making technique, is used to optimize the level of police services. Under this technique, the strategic spatial distribution of post/patrol locations are determined using the idea of *Maximum Coverage Location Problem (MCLP)* solved by Binary Integer Linear Programming. Obtaining *minimum-size dominating sets* for network clustering is used, either to create police districts, or to partition areas of mobile patrol units. Moreover, an algorithm is formed to schedule an eight-hour shift of police officers. Deterministic models are used in this study.

A prerequisite to the implementation of the Maximum Coverage Location Model is the construction of the community network. A typical police zone is divided into smaller regions called *districts*. Districts are further subdivided into atomic regions. An *atom* is determined by a circular region with *radius* ***R*** and a center, called *centroid*. However, it should be noted that radius ***R*** is different from the radius of a graph as defined in Graph Theory.

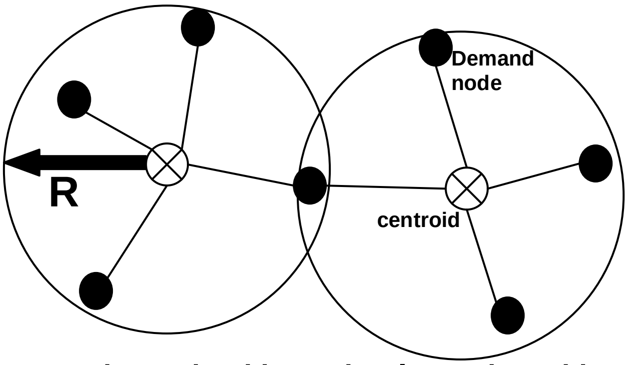

Figure 1. Atoms determined by a circular region with radius R.

Church and Revelle (1974) first introduced the Maximum Coverage Location Problem, which is a combinatorial optimization problem. Given a simple and symmetric network $G=(V,E)$ and positive parameters P_m and R_t , the task of MCLP is to find S subset of V, where $|S| \leq P_m$, such that S maximizes the number of covered vertices whose distance from S is at most R_t (Mihelič and Robič, nd.). In other words, the objective of MCLP is to maximize the number of incidents that can be served within a given service distance (or within a corresponding desired response time). The following Goal Program is a modified version of the traditional MCLP Binary Integer Linear Program formulation of Long Ma (2003):

$$\textit{Maximize Coverage} = \sum_{i \in I} w_i y_i \quad \textit{while Minimizing } P_m$$

subject to

$$\textit{Constraint } 1\text{:} \sum_{j \in Z_i} x_j \geq y_i \text{ , } \textit{for all } i \in I$$

$$\textit{Constraint } 2\text{:} \sum_{j \in J} x_j = P_m$$

$$\textit{Constraint } 3\text{: } \textit{For all } j \in J$$

$$x_j = \begin{cases} 1 \text{ , } \textit{police officer is allocated to node } j \\ 0 \text{ , } \textit{otherwise} \end{cases}$$

$$\textit{Constraint } 4\text{: } \textit{For all } i \in I$$

$$y_i = \begin{cases} 1 \text{ , } \textit{one or more police officers are located at the centroids in } Z_i \\ 0 \text{ , } \textit{otherwise} \end{cases}$$

where;

- ☞ I is the set of buildings or places to be covered;
- ☞ J is the set of potential police post/patrol area centroids;

- ☞ R_t is the desired service distance based on the desired response time;
- ☞ d_{ij} is the shortest distance from node $i \epsilon I$ to $j \epsilon J$;
- ☞ w_i is the weight of node $i \epsilon I$;
- ☞ P_m is the number of police officers to be spatially located (in a district); and
- ☞ Z_i is the set of police post/patrol area centroids that are eligible to provide cover to demand node $i \epsilon I$. $Z_i = \{j \epsilon J \mid d_{ij} \leq R_t\}$.

The model intends to yield an efficient solution, possibly a compromise solution based on the relative importance of each conflicting goal (i.e., maximizing coverage versus minimizing number of police officers). The goal $Minimize\ P_m$ is synonymous to Constraint 2 when changed to $\sum_{j \in J} x_j = P_m - n, for\ n = 0, 1, 2, \ldots, P_m - 1$. This aims to obtain the number of police officers less than the originally computed P_m, which can still cover the nodes covered by the originally computed P_m. This intends to minimize the cost of hiring additional police officers. Moreover, if node i is covered with two or more centroids, then the nearest centroid will be taken into account unless that centroid has many workload.

There are two ways to compute for the number of police officers. They are as follows:

Method 1: (maximum coverage is the priority)

$$P_m = \left\lfloor \frac{A_v}{A_d} + P_f \right\rfloor$$

$$A_d = \pi R_t^{\ 2}$$

$$P_T = S \bullet (P_m\ of\ district\ 1 + P_m\ of\ district\ 2 + P_m\ of\ district\ 3 + \cdots + P_m\ of\ district\ k + P_a)$$

where;

- ☞ A_v is the area of the district in consideration;
- ☞ A_d is the desired area of an atom;
- ☞ P_f is the number of police officers in predetermined fixed posts (such as guards at the gate);
- ☞ P_T is the total number of police officers in the community;
- ☞ k is the number of districts formed;
- ☞ P_a is the number of additional police officers such as mobile unit drivers, desk officer, team leader, etc.; and
- ☞ S is the number of shifts per day.

Method 2: (satisfying the budget constraint is the priority)

$$P_T = \left\lfloor \frac{B_u}{B_s} \right\rfloor$$

$$Area\ of\ atom = \frac{A_T}{\left(\frac{P_T}{S} - P_a - P_f\right)}$$

$$Radius\ of\ atom = \sqrt{\frac{Area\ of\ atom}{\pi}}$$

$$P_m = \left\lfloor \frac{A_v}{Area\ of\ atom} \right\rfloor$$

where;

- ☞ A_T is the total area of the whole community that needs police coverage;
- ☞ B_u is the total allocated budget for the salaries and wages of police officers;
- ☞ B_s is the amount of salaries and wages of a police officer;
- ☞ P_T is the total number of police officers in the community;
- ☞ S is the number of shifts per day;
- ☞ P_a is the number of additional police officers such as mobile unit drivers, desk officer, team leader, etc.;
- ☞ P_f is the number of police officers in predetermined fixed posts (such as guards at the gate); and
- ☞ A_v is the area of the district in consideration.

Reducing personnel fatigue by relaxing their hectic schedules may lead to hiring additional police officers. The computation of P_m should come hand in hand with the derived schedule. The construction of the eight-hour shift schedule of police officers considers the following assumptions:

- ☞ Police personnel must work eight hours a day and, if possible, with two consecutive vacations per week;
- ☞ The schedule must be rotational for 24 hours and 7 days a week;
- ☞ A police officer must have at least 16 hours rest before returning back to work; and
- ☞ Equity among personnel must be satisfied (e.g. equally getting opportunity of having vacation during Saturdays and Sundays).

Since the least common multiple of the number of shift per day and the number of days per week or *LCM(4,7)* is equal to 28 days, therefore the schedule will be created for 28 days or four weeks. The following table shows the recommended schedule format:

Table 1. Generated rotational schedule for four weeks.

1st Week	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Shift 1	*T4	T4	T3	T3	T2	T2	T2
Shift 2	T1	T1	T4	T4	T3	T3	T3
Shift 3	T2	T2	T1	T1	T1	T4	T4
Vacation	T3	T3	T2	T2	T4	T1	T1

Table 1. Generated rotational schedule for four weeks. (continuation)

2nd Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Shift 1	T1	T1	T4	T4	T3	T3	T3
Shift 2	T2	T2	T1	T1	T4	T4	T4
Shift 3	T3	T3	T2	T2	T2	T1	T1
Vacation	T4	T4	T3	T3	T1	T2	T2

3rd Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Shift 1	T2	T2	T1	T1	T4	T4	T4
Shift 2	T3	T3	T2	T2	T1	T1	T1
Shift 3	T4	T4	T3	T3	T3	T2	T2
Vacation	T1	T1	T4	T4	T2	T3	T3

4th Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Shift 1	T3	T3	T2	T2	T1	T1	T1
Shift 2	T4	T4	T3	T3	T2	T2	T2
Shift 3	T1	T1	T4	T4	T4	T3	T3
Vacation	T2	T2	T1	T1	T3	T4	T4

**T means Team*

Model Assessment

The models are tested using the structure of the University of the Philippines (UPLB) community. This model assessment is based from the study of Rabajante, et al. (2005). In fact, the models are generalizations of that study. The models are generalized so that various communities could apply the models to their respective situation.

The assumptions used are:

- Frame of reference is Euclidean geometric distance.
- The weights of the edges of Network $G=(V,E)$ are distances. An edge is an element of E, if and only if the distance between the nodes connected by that edge is less than or equal to the desired service distance.
- The desired service distance, R_t, is set at 150m. This comes from the assumption that the lead time of emergency response is five minutes.
- There are two mobile patrol units to be deployed. 'Blue guards' are also considered in the study.
- The scenarios used are based on year 2005. The amount of budget is confidential in nature and will not be presented in this paper.

The following procedures are performed:

1. The UPLB campus is partitioned into five districts, and the campus map is converted into a graph/network;
2. The possible demand nodes and police post centroids are selected;
3. P_m's are computed using Methods 1 and 2;

4. The MCLP is solved using WinQSB version 1;
5. The eight-hour shift schedule is derived using the recommended format;
6. Minimum-size dominating sets are determined to distribute the mobile patrol units; and
7. The efficient number of police personnel is determined given the limited budget.

The following are the results derived from Methods 1 and 2:

Table 2. Comparison of the results using Method 1 and Method 2.

	Method 1 (maximum security)	**Method 2 (with budget constraint)**
P_T (total number of police officers)	100	60
P_m for Pili Drive district	4	2
P_m for Lower Campus district	8	5
P_m for UPCO district	3	2
P_m for CVM district	2	1
P_m for CFNR district	4	2
Radius of the atom (in meters)	150	255.14

The following table compares the result of Method 2 and the situation during year 2005:

Table 3. Comparison of results using Method 2 and the scenario in year 2005.

	Method 2 (8-hour shift)	**Year 2005 Scenario (12-hour shift)**
P_T (total number of police officers)	60	36
Number of police per shift	15	12
P_m for Pili Drive district	2	1 + ½*
P_m for Lower Campus district	5	5
P_m for UPCO district	2	1
P_m for CVM district	1	½*
P_m for CFNR district	2	1
Radius of the atom (in meters)	255.14	≤404.17
Approximated worst-case response time (in minutes)	13	20
Number of uncovered nodes	44	53

**CVM and Pili Drive area share one police officer.*

Increasing patrol visibility could help frighten possible lawbreakers and it can raise public safe feeling. See Appendix A for the maximum coverage posts of police officers using Method 2. College of Forestry and Natural Resources (CFNR) district is subdivided into four sectors, since the nodes are considerably distant to each other.

Two patrol zones are determined by finding the minimum-size dominating sets in the network. Two mobile patrol units will be deployed in these zones.

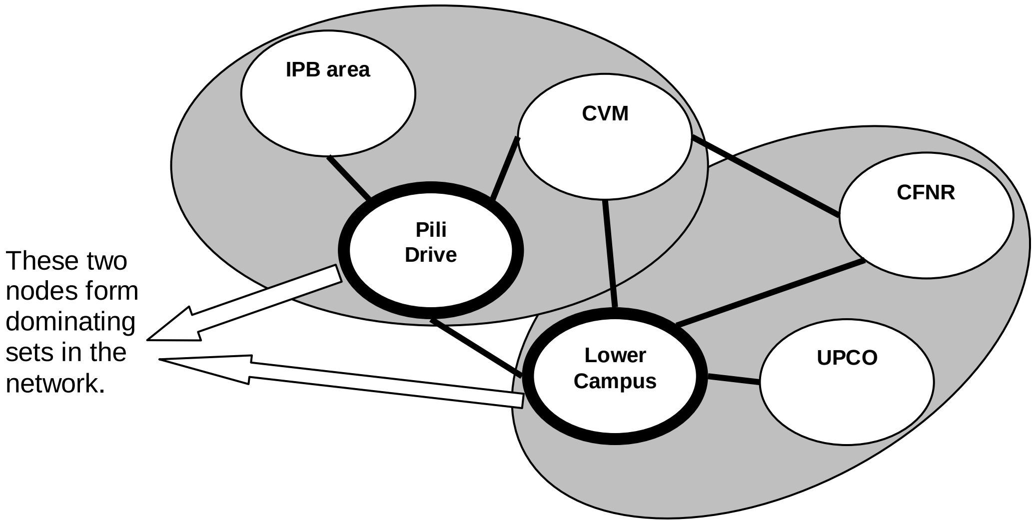

Figure 2. The five districts with IPB area partitioned into two patrol zones.

Concluding Remarks

At the end of every Operation Research methodology, the decision-maker should verify the result. The models presented are notionally applied to the structure of the UPLB community. The policymakers in various communities can use the generalized models to optimize their police force deployment. In fact, the models are suitable not only to police force deployment, but also to the allocation of other types of security personnel, such as the contractual guards.

It is more recommended to use Network distance than Euclidean distance to obtain an accurate measure of distances. The use of Geographic Information System would improve the digitization of maps to graphs/networks. Moreover, the decision-maker still has the opinion if other shifting schedule will be applied, since the mandated eight-hour shift schedule necessitates a significant increase in the number of personnel.

Deterministic models are utilized for planning purposes. However, the models can be extended for operational scenarios by adding stochastic and ‘fuzzy’ elements of police deployment. Maximum Coverage Location Problem can also be used in locating other emergency and service facilities such as fire stations, hospitals, and public markets.

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Software: WinQSB. Version 1 for Windows. Copyright Yih-Long Chang.

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APPENDIX A

Optimal UPLB Police Force Posts Using Method 2

	CENTROID	COVERED DEMAND NODES
PILI DRIVE AREA		
Police 1	AMTEC	Experimental Fields, Biological Control Lab., National AgroMet Station, AMDP, Old Engineering Bldg.
Police 2	CEAT	Fruit Crops Orchard, PhilRice, Experimental Field, Old Agronomy Bldg., ASH, SPMO, Gasoline Station, UPLB Central Parking Area, CPDMO, Chem. Engineering Bldg., IFST, Post Harvest
LOWER CAMPUS		
Police 1	Admin Bldg.	
Police 2 and 3	Main Gate	
Police 4	NCAS	CEM Secretary's Office, BioSci Bldg., Carabao Park, CDC Bldg., DZLB Tower, ACCI, CEM Dean's Office, OVCCA and Department of AgEcon, Department of Economics, Department of AgriBusiness, Raymundo Gate, SESAM, DAERS, Old PLDT Bldg.,CHE, Greenhouse and Headhouse (BioSci), OUR, NCAS, SEARCA, COOP, Humanities Bldg., Oblation, Old Chem. Bldg.
Police 5	ACCI Dorm	Graduate School, Business Affairs Office, DL Umali Auditorium, Public Toilet, Freedom Park, Track and Field and Soccer Field, Grandstand, YMCA, Swimming Pool, Baker Hall, Basketball and Tennis Court, DMST, Palma Bridge, Hortorium, SU, Women's Dorm, IH Dorm, Men's Dorm, Pahinungod
UPF Headquarters	UPF Headquarters	PhySci Bldg., Nutrition Bldg., Ornamental Crops Nursery, Ornamental Crops Division, Fruit Crops Nursery, Post Office, UPLB Foundation, Senior Social Garden, Home Tech. and Child Development Lab., Math Bldg.
UPCO		
Police 1	Executive House	
Police 2	Guard House	UPCO Staff Housing, Substation, Doña Aurora St. Entrance/Exit, Sampaguita St. Entrance/Exit, Jasmin St. Entrance/Exit, Gumamela St. Entrance/Exit
CVM		
Police 1	CVM Lab. Bldg.	NCPC Dorm, NTC, RTP-FNP, UPLB Corral, CEC, Old Animal Husbandry Bldg., GYM, UPLB Medicinal Plants Garden and Gene Bank, PCRDF, CVM Admin. and Office of College Secretary, IAS Villegas Hall (Admin. Bldg.), IAS Fronda Hall, Vet. Med. Library (Communal Bldg.)
CFNR		
Police 1	CPAF	UHS, College Country Club, SEARCA Four-Door Apartments, IAS-CVM Housing
Police 2 (sector 3)	Morning: FPRDC-Forest Products Lab. Bldg., Pulp and Paper Research Training Center, Treatment Plant and Shop Night: MAREHA	FPRDC, MAREHA, Makiling Botanical Garden, Wood Science and Tech. Bldg., Wood Chemistry and Physics Research Lab., CFNR Admin. Bldg., Forestry Biological Science Bldg., ERDB, UPLB Museum of Natural History, Institute of Forest Conservation, Forestry Info. and Library Bldg., FOREHA, New FOREHA, Sawmill, Heavy Equipment Garage, Forest Reserve Entrance/Exit, Forestry Alumni Guesthouse, Tennis Court/Soccer Field, Makiling Heights Housing UPCF, CF-IFC Guesthouse 1, 2 and 3, Forestry COOP and Canteen