P-NOYmial Interpolation

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ABSTRACT

Interpolation is the approximation of points between given surrounding points or values. It is a method in which functions including its graph are generated using the given points. It has various applications such as photo editing and weather forecasting. We used interpolation for fun, artistic and creative purposes. Having said this, we chose a photo of a prominent public figure and created a representation of it using the polynomial interpolation method.

Benigno "P. Noy" Aquino III, the newly elected president of the Philippines, signifies new found hope, inspiration, and a chance for a new start for a country longing for change. Our group came up with a mathematical representation of our very own president's picture to serve as an example on how interpolation works.

METHODOLOGY

⇒ Get an image of President Noynoy's face and place it on a graphing sheet
⇒ Select sets of control points for each of the facial features, e.g. nose, eyes and glasses.
⇒ Use the selected points to interpolate the curves that will best represent the given feature.
⇒ Use appropriate interpolation methods, such as Newton's Divided Difference and Bézier curves.
⇒ Graph all the generated polynomials in a single *xy*-plane (MS Excel can be used).

Polynomial Interpolation

In parametric polynomial interpolation, the first step is to separate the x and y components of the control points. Next, pair up a set of t components to each of the corresponding x and y components. Then, the Newton's Divided Difference method is used for obtaining the usual interpolating polynomial. The algorithm for Newton's Divided Difference follows the recursive property:

$$f(t_0,t_1,...,t_k)=\frac{f(t_1,t_2,...,t_k)-f(t_0,t_1,...,t_{k-1})}{t_k-t_0},0\le t\le 1$$

The usual interpolating polynomial P(x) can be obtained from the equation:

$$P(t)=a_0+\sum_{i=1}^{k}\left[a_i\prod_{j=0}^{i-1}(t-t_j)\right],0\le t\le 1$$

On the other hand, to get the interpolating Bézier curves, we need sets of Bezier points.

$$P(t)=\begin{bmatrix}x(t)\\y(t)\end{bmatrix},\ 0\le t\le 1$$

The Bézier interpolating function follows the Bernstein basis polynomials of the form:

$$\sum_{i=0}^{n}\binom{n}{i}(1-t)^{n-i}t^i p_i$$

RESULTS AND DISCUSSION

Figure 2 is the collection of the generated interpolated polynomials. For most sections, Newton's Divided Difference method was used compared to the Bézier curves. Most of the facial features are continuous and composed of long curves in which the usual polynomial interpolation is more advantageous to be used; but in some parts where the lines are shorter and where there are more curves, the Bézier curves are more appropriate to be used than the usual polynomial interpolation.

FIGURE 1. Photo of President Aquino.

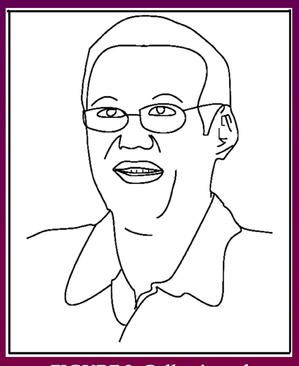

FIGURE 2. Collection of Interpolated Polynomials.

FIGURE 3. Compilation of Polynomial graphs for the eyeglasses

Interpolating polynomials for the eyeglasses

Left Lens
x(t)=-(-6.8)+(2.8)(t-0)+(0)(t-0)(t-1)+(-.566667)(t-0)(t-1)(t-2)+(.216667)(t-0)(t-1)(t-2)(t-3)+(-.043333)(t-0)(t-1)(t-2)(t-3)(t-4)+(.006111)(t-0)(t-1)(t-2)(t-3)(t-4)(t-5)
y(t)=-(5.5)+(.5)(t-0)+(-.8)(t-0)(t-1)+(.133333)(t-0)(t-1)(t-2)+(.058333)(t-0)(t-1)(t-2)(t-3)+(-.033333)(t-0)(t-1)(t-2)(t-3)(t-4)+(.011111)(t-0)(t-1)(t-2)(t-3)(t-4)(t-5)

Right Lens
x(t)=-(.4)+(2.6)(t-0)+(.2)(t-0)(t-1)+(-.65)(t-0)(t-1)(t-2)+(.225)(t-0)(t-1)(t-2)(t-3)+(-.036667)(t-0)(t-1)(t-2)(t-3)(t-4)+(.0025)(t-0)(t-1)(t-2)(t-3)(t-4)(t-5)
y(t)=-(5.5)+(1)(t-0)+(-1.1)(t-0)(t-1)+(.183333)(t-0)(t-1)(t-2)+(.066667)(t-0)(t-1)(t-2)(t-3)+(-.0375)(t-0)(t-1)(t-2)(t-3)(t-4)+(.01111)(t-0)(t-1)(t-2)(t-3)(t-4)(t-5)

Bridge
x(t)=-(-1.4)+(.9)(t-0)+(.05)(t-0)(t-1)
y(t)=-(4.8)+(.2)(t-0)+(-.2)(t-0)(t-1)

Stand
x(t)=-(5.8)+(4.7)(t-0)+(-2.25)(t-0)(t-1)
y(t)=-(5.5)+(1)(t-0)+(-1.15)(t-0)(t-1)

CONCLUDING REMARKS

Two interpolating techniques were used, namely polynomial interpolation using divided difference and Bézier curves, to model the facial features of President Benigno "Noynoy" Aquino III. Each of the method mentioned above has its own advantages and disadvantages.

There were certain observations encountered while interpolation of the selected points are being performed. "Polynomial wiggles" occur when too many points are being considered in the interpolation. But sometimes, many points yield to a better graphical representation. Bézier curve is good for graphs that contain short and curvy segments. Usual polynomial interpolation (using divided difference) is good for continuous behavior of graph which contains long smooth curves.

As a recommendation for future work, others may try other interpolation techniques and compare the result with our result. Also, others may try to use smaller intervals for interpolation to obtain an interpolating polynomial that resembles the facial features more closely.

Reference:

Burden RL and Faires JD (2005). *Numerical Analysis*. 8th ed. Thomson Brooks/Cole.

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